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# Eddy Viscosity-Entrainment Correlation

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## Introduction

THERE are several types of inverse methods to obtain solutions for separated boundary layer. Specifying the distributions of a boundary-layer property (the displacement thickness or the skin friction coefficient) and calculating the corresponding velocity distributions at the outer edge of the boundary layer, these methods eliminate the singularity in the boundary-layer direct methods, at separation.

Inverse approaches for two-dimensional flows using the integral methods have been developed from 1973 onward.[1] In this note, the streamwise momentum integral equation and the entrainment equation[2] are used to describe the pressure-driven separated boundary-layer development in conjunction with the Horton's lag-entrainment equation,[3] and an assumed eddy viscosity-entrainment correlation for equilibrium boundary layers.

## Basic Equation

Among the most widely used and successful of many models available for the prediction of turbulent boundary-layer development are versions of the basic entrainment method conceived by Head.[2]

In Head's original scheme, the equations to be integrated are the momentum integral equation

$$\frac{\mathrm{d}}{\mathrm{d}x}(U_e\theta) = \frac{1}{2}C_fU_e - (H+1)\theta\frac{\mathrm{d}U_e}{\mathrm{d}x} \tag{1}$$

(for two-dimensional, constant-density flow) and the entrainment equation which for two-dimensional, constant-density flow can be written in the forms

$$V_E = U_eC_E = \frac{\mathrm{d}}{\mathrm{d}x}\int_0^\delta U\,\mathrm{d}y = \frac{\mathrm{d}}{\mathrm{d}x}[U_e(\delta - \delta^*)]$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}(U_e\theta H_1) \tag{2}$$

In the preceding equations, $U_e$ is the mean velocity at the edge of the boundary layer, $C_f$ is the local skin friction coefficient, $U$ is the time-mean velocity within the boundary layer of thickness $\delta$, so that $U_e = U|_{y=\delta}$ and $C_E$ is the entrainment coefficient. The momentum and displacement thicknesses are denoted by $\theta$ and $\delta^*$, respectively, and $H = \delta^*/\theta$, $H_1 = (\delta - \delta^*)/\theta$ are the common shape factor and the mass-flow shape parameter, respectively. The entrainment velocity $V_E$ and the entrainment coefficient $C_E$ are related by $C_E = V_E/U_e$.

The strong history effects characteristic to turbulent boundary layers subject to rapid changes in the streamwise pressure gradient are modeled by Horton's[3] rate equation for $C_E$, which is actually a simplified form of the stress-transport equation of Bradshaw[4]

$$\frac{\mathrm{d}C_E}{\mathrm{d}x} = \frac{1}{2\delta}(C_{E_{\mathrm{eq}}} - C_E) \tag{3}$$

$C_{E_{\mathrm{eq}}}$ represents the entrainment coefficient in the equilibrium boundary layers which will be discussed in the following section.

In addition to the basic Eqs. (1), (2), and (3), some auxiliary equations are necessary. These auxiliary equations are based on the Coles' velocity profiles in an extended form.[1] The displacement thickness $\delta^* = \delta^*(x)$ was chosen and then input to the inverse boundary-layer problem. Two ordinary differential Eqs. (1) and (2), and two algebraic equations derived from the velocity profile are available for the solution. This system of coupled differential equations is solved by a fractional step-integration scheme. The uncoupled lag-entrainment Eq. (3) is solved separately.

## Review of Existing Correlations

The entrainment coefficient $C_E$, is defined as the (nondimensional) rate at which fluid from external inviscid flow enters through the outer edge of the boundary layer.

A first attempt to quantify this effect was performed by Head in 1958. In Head's original method,[2] and in Green's later extension of it to compressible flow,[5] $C_E$ was defined empirically as a function of $H_1$

$$C_E = C_{E_{\mathrm{eq}}} = 0.03(H_1 - 3.0)^{-0.617} \tag{4}$$

and respectively

$$C_{E_{\mathrm{eq}}} = H_1\{0.0302[(H+1)(H-1)^2/H^3]$$
$$- \tfrac{1}{2}C_f[0.25 + (1.25/H)]\} \tag{5}$$

In 1967, based on a reduced form of the turbulent energy equation, "diffusion" = "advection," Bradshaw[4] established the relationship

$$C_E = 10(\tau_{\max}/\rho U_e^2)^{1.0} \tag{6}$$

where $\tau_{\max}$ designates the maximum shear stress. This correlation was assumed to be held both in equilibrium and nonequilibrium boundary layers, and even in the mixing layer.

In 1969, Horton, in a lag-entrainment method,[3] suggested that $C_{E_{\mathrm{eq}}}$ can be directly related to the mean shear stress of the outer part of the boundary layer by the relationship

$$C_{E_{\mathrm{eq}}} = 1.585(\tau_{1/2}/\rho U_e^2)^{0.69} \tag{7}$$

where $\tau_{1/2}$ is the shear stress at $y/\delta = 0.5$. It is computed with the Cebeci-Smith turbulence model[6] for the outer layer which is highly inaccurate for equilibrium layers in strong adverse pressure gradient.

Head and Galbraith[7] use a different approach in determining $C_{E_{\mathrm{eq}}}$. They started by using the general entrainment definition, as the rate at which fluid is crossing a line of constant $U/U_e$, and at the first boundary layer this becomes

$$C_E = \frac{1}{U_e}\lim_{y\to\infty}\left(\frac{\dfrac{1}{\rho}\dfrac{\partial\tau}{\partial y}}{\dfrac{\partial U}{\partial y}}\right) \tag{8}$$

Received Dec. 16, 1991; revision received April 20, 1992; accepted for publication May 7, 1992. 

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Then using an eddy viscosity expression for $\tau$ in conjunction with Thomson's family of velocity profiles, and assuming that the eddy-viscosity profiles are similar, they derived the correlation

$$C_{E_{eq}} = 7.2(\delta^*/\delta)[\nu_{\tau_{max}}/(U_e\delta^*)]_{eq} \quad (9)$$

According to Ref. 8, for any equilibrium layer characterized by a particular value of the velocity-defect parameter $G\,[=(H-1)/H(2/|C_f|^{1/2})]$, $(\nu_{\tau_{max}}/U_e\delta^*)_{eq}$ can be represented analytically by

$$[\nu_{T_{max}}/(U_e\delta^*)]_{eq} = 0.002094 + 0.02672(1 - e^{-0.1163G}) \quad (10)$$

In conjunction with Eq. (8) for equilibrium flows, by using a particular form of the velocity profile $u(\eta)$ (where $u = U/U_e$ and $\eta = y/\delta$), Le Balleur[9] derived the expression (with empirical constants)

$$C_{E_{eq}} = 0.118(\delta^*/\delta)/(1 + 1.22\eta^*) - 0.018(C_f/|C_f|^{1/2}) \quad (11)$$

here $\eta^* = 0$ for attached or slight separated flow. For large values of $H$, both $\delta^*/\delta$ and $\eta^*$ approach 1 and $C_f \to 0$, so that $C_{E_{eq}} \to 0.053$, the value of which does not agree with the only experimental measurements of a separated equilibrium flow ($C_E \simeq 0.067$ when $H = 5.5$).

## Proposed Correlation

Since the above correlation fails in the calculation of separated flow, we propose a correlation suitable for separated equilibrium flow. We tried to improve the formula [Eq. (8)] because of its more rational content. With a view to improvement, we make the reasonable further assumption of replacing the similarity of the eddy viscosity profile; similarly an assumption of a conjecture on the slope variation of the eddy viscosity distribution at the boundary-layer edge is also proposed. According to this conjecture, the above quantity is approximated by the following expression:

$$\lim_{y\to\delta}\left[\frac{\partial(\nu_T/\nu_{T_{max}})}{\partial(y/\delta)}\right] = -\left(\frac{\nu_{T_{max}}}{U_e\delta^*}\right)_{eq}^{-0.43} \quad (12)$$

Introducing this relation into Eq. (8), we can obtain for entrainment coefficient a correlation of form

$$C_{E_{eq}} = (\delta^*/\delta)[\nu_{T_{max}}/(U_e\delta^*)]_{eq}^{0.57} \quad (13)$$

Actually, the correlation method was compared with measurements in a two-dimensional separating turbulent boundary layer. The figures below show the boundary-layer properties $\delta^*$, $H$, $C_f$, and $U_e$ of the Simpson et al.[10] experiment (Fig. 1), and the Hastings'[11] experiment (Fig. 2), and compares them with the calculus.

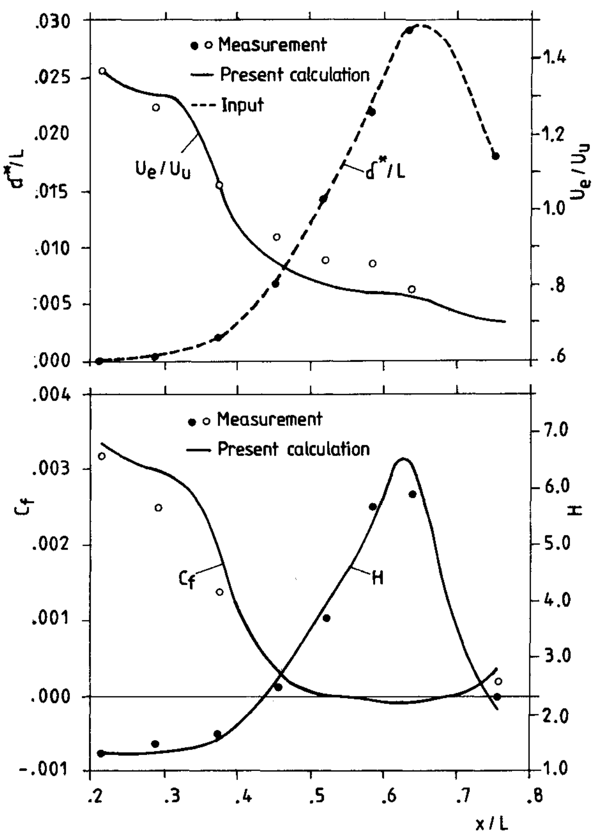


**Fig. 1 Measured[10] and computed boundary-layer parameters in low-speed diffuser.**

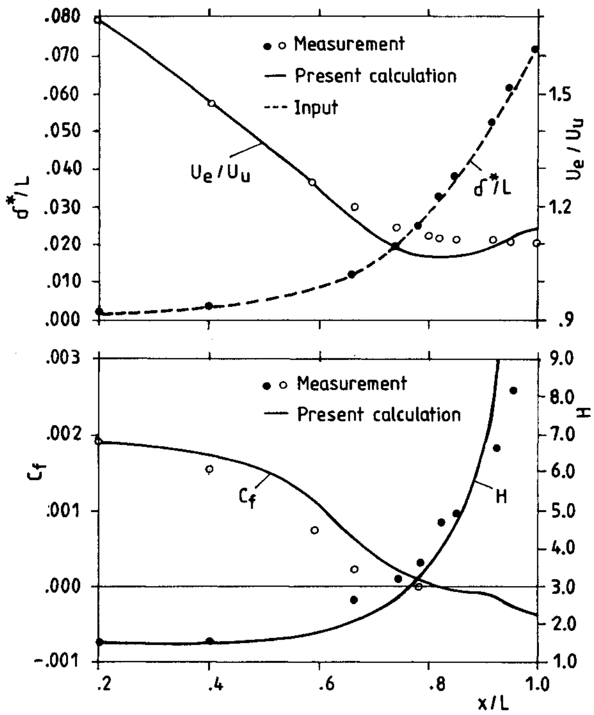


**Fig. 2 Measured[11] and computed boundary-layer parameters in NACA 4412 airfoil flowfield.**

## Summary

A correlation for entrainment velocity has been formulated specifically for equilibrium two-dimensional, turbulent boundary-layer flows near separation.

The proposed correlation has been incorporated into the inverse boundary-layer integral method based on the lag-entrainment method of Horton. The agreement between the calculations and experiments in two-dimensional incompressible flow is quite acceptable, even for larger separated flow regions.

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# Method for Assessing the Electric Power System Reliability of Multiple-Engined Aircraft

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## Introduction

THE reliability of an electricity generating system is an essential issue in the design analysis of most aircrafts.[1,2] Adequate electricity power supply is critical for successful operations of airborne engines, control surfaces, radar, weaponry, instruments, etc. In an attempt to achieve a high level of reliability for this important airborne electric power system, two or more electric generators are usually arranged in parallel to be driven by each engine of an aircraft as shown in the example for a twin-engine aircraft (Fig. 1), where each electricity generator has a power capacity of 30 kVA. Assuming that with a minimum electric power supply of 30 kVA the aircraft can barely maintain flight in an emergency operation (case A). On the other hand, if the electricity generators together can supply a minimum of 60 kVA, then the aircraft can operate as normal (case B).

To assess the system reliability[3,4] of such a typical aircraft, electricity generating system is important in the design analysis phase of aircraft. For a standard flight session if each electricity generator has a reliability rated at $L_{Gi}$ (where the subscript $i$ represents the designated generator number), then the corresponding failure rate $F_{Gi}$ is equal to $(1\text{-}L_{Gi})$. The reliability and failure rate for each engine are denoted as $L_{Ei}$ and $F_{Ei}$, respectively. Consequently, the probability of success for emergency electricity supply (case A) can be easily expressed as

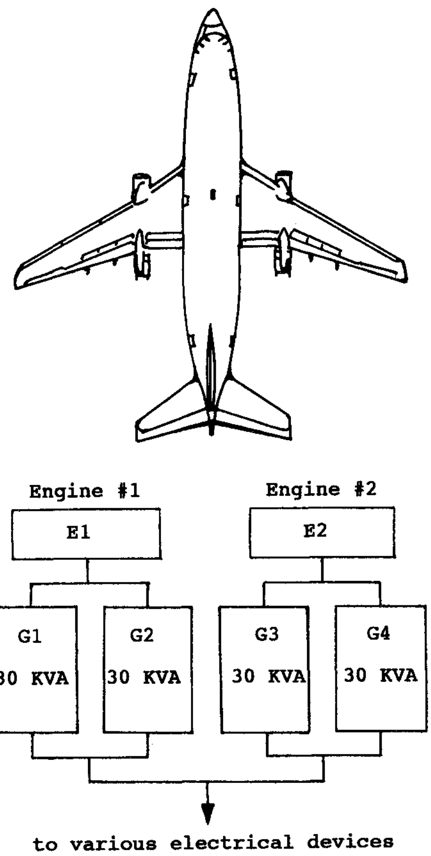


**Fig. 1 Twin-engine four-generator system.**

$$[(P_{\text{success}})_{\text{syst}}]_{30\,\text{kVA}} = 1 - \{1 - [1 - (F_{G1})(F_{G2})]L_{E1}\} \cdot \{1 - [1 - (F_{G3})(F_{G4})]L_{E2}\} \tag{1}$$

But the logical reasoning to obtain the expression of system reliability for the normal power supply (case B) is rather complicated, usually the traditional method with simplex conditional probability concept is employed to attack this problem. In that case, the system reliability can eventually be expressed as

$$[(P_{\text{success}})_{\text{syst}}]_{60\,\text{kVA}} = 1 - \{[((1 - L_{G2})\{1 - [L_{E2}(1 - F_{G3}F_{G4})]\}L_{E1}) + (1 - L_{E2}L_{G3}L_{G4})F_{E1}]L_{G1} + \{[1 - L_{E2}(1 - F_{G3}F_{G4})]L_{E1}L_{G2} + (1 - L_{E2}L_{G3}L_{G4}) \cdot (1 - L_{E1}L_{G2})\}F_{G1}\} \tag{2}$$

The whole reasoning process becomes even more difficult for more complicated cases with more engines and associated electric generators. In order to solve this dilemma, a combinational pivotal decomposition method (CPDM) is developed to assess the system reliability of the aircraft electricity generating system in a much more efficient and systematic manner, both in ideology and implementation, in contrast to the traditional conditional probability methodology. Several typical examples are utilized below to illustrate the necessary definitions, assumptions, concepts, and detailed mathematical procedures.

Received Oct. 17, 1991; revision received April 15, 1992; accepted for publication May 20, 1992. 

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